LESSON 5. 4a

Solving Radical Equations

Today you will:

- Solve equations containing radicals.
- Find extraneous solutions.
- Practice using English to describe math processes and equations

Basic process of solving an equation

1. Isolate the variable
   - What's with the variable?
   - How is it combined?
   - How do I undo it?
2. Keep the balance
   - Whatever you do to one side you must do to the other
3. Simplify your answer
   - Simplify inside radical
   - No negative exponents
   - No fractions in radicals
   - No radicals in denominator
   - $\pm$ if and only if solving equation and start with even root
4. Check your answer
   - Plug answers back into original equation, look for extraneous solutions

Don't forget PEMDAS!

- Parenthesis
- Exponents
- Multiplication/division (left to right)
- Addition/subtraction (left to right)

What is an extraneous solution?

Raising each side of an equation to the same exponent may introduce solutions that are not solutions of the original equation.

Solve (a) $2\sqrt{x+1} = 4$ and (b) $\sqrt[3]{2x-9} - 1 = 2$.

SOLUTION

a.

$2\sqrt{x+1} = 4$	Write the original equation.
$\sqrt{x+1} = 2$	Divide each side by 2.
$(\sqrt{x+1})^2 = 2^2$	Square each side to eliminate the radical.
$x + 1 = 4$	Simplify.
$x = 3$	Subtract 1 from each side.

▶ The solution is $x = 3$.

Check

$2\sqrt{3+1} \stackrel{?}{=} 4$

$2\sqrt{4} \stackrel{?}{=} 4$

$4 = 4$ ✓

b.

$\sqrt[3]{2x-9} - 1 = 2$	Write the original equation.
$\sqrt[3]{2x-9} = 3$	Add 1 to each side.
$\left(\sqrt[3]{2x-9}\right)^3 = 3^3$	Cube each side to eliminate the radical.
$2x - 9 = 27$	Simplify.
$2x = 36$	Add 9 to each side.
$x = 18$	Divide each side by 2.

▶ The solution is $x = 18$.

Check

$\sqrt[3]{2(18)-9} - 1 \stackrel{?}{=} 2$

$\sqrt[3]{27} - 1 \stackrel{?}{=} 2$

$2 = 2$ ✓

Solve the equation. Check your solution. $\sqrt[3]{x} - 9 = -6$

SOLUTION

a. $\sqrt[3]{x} - 9 = -6$ Write the original equation.

$\sqrt[3]{x} = 3$ Add 9 to each side.

$(\sqrt[3]{x})^3 = 3^3$ Cube each side to eliminate the radical.

$x = 27$ Simplify.

▶ The solution is $x = 27$.

Check

$\sqrt[3]{27} - 9 \stackrel{?}{=} -6$

$3 - 9 \stackrel{?}{=} -6$

$-6 = -6$ ✓

Solve the equation. Check your solution. $\sqrt{x+25} = 2$

SOLUTION

a. $\sqrt{x+25} = 2$ — Write the original equation.

$(\sqrt{x+25})^2 = 2^2$ — Square each side to eliminate the radical.

$x + 25 = 2$ — Subtract 25 from each side.

$x = -21$

Check

$\sqrt{-21+25} \stackrel{?}{=} 2$

$\sqrt{4} \stackrel{?}{=} 2$

$2 = 2$ ✓

▶ The solution is $x = -21$.

Solve the equation. Check your solution. $2\sqrt[3]{x-3} = 4$

SOLUTION

a. $2\sqrt[3]{x-3} = 4$ Write the original equation.

$\sqrt[3]{x-3} = 2$ Divide each side by 2.

$(\sqrt[3]{x-3})^3 = 2^3$ Cube each side to eliminate the radical.

$x - 3 = 8$ Add 3 to both sides.

$x = 11$

▶ The solution is $x = 11$.

Check

$2\sqrt[3]{11-3} \stackrel{?}{=} 4$

$2\sqrt[3]{8} \stackrel{?}{=} 4$

$2 \cdot 2 = 4$ ✓

In a hurricane, the mean sustained wind velocity v (in meters per second) can be modeled by $v(p) = 6.3\sqrt{1013 - p}$, where p is the air pressure (in millibars) at the center of the hurricane. Estimate to 1 decimal place the air pressure at the center of the hurricane when the mean sustained wind velocity is 54.5 meters per second.

SOLUTION

$v(p) = 6.3\sqrt{1013 - p}$	Write the original function.
$54.5 = 6.3\sqrt{1013 - p}$	Substitute 54.5 for $v(p)$.
$8.65 \approx \sqrt{1013 - p}$	Divide each side by 6.3.
$8.65^2 \approx (\sqrt{1013 - p})^2$	Square each side.
$74.8 \approx 1013 - p$	Simplify.
$-938.2 \approx -p$	Subtract 1013 from each side.
$938.2 \approx p$	Divide each side by -1.

▶ The air pressure at the center of the hurricane is about 938.2 millibars.

Solve $x + 1 = \sqrt{7x + 15}$.

SOLUTION

$x + 1 = \sqrt{7x + 15}$	Write the original function.
$(x + 1)^2 = (\sqrt{7x + 15})^2$	Square each side.
$x^2 + 2x + 1 = 7x + 15$	Expand left side and simplify right side.
$x^2 - 5x - 14 = 0$	Write in standard form.
$(x - 7)(x + 2) = 0$	Factor.
$x - 7 = 0$ or $x + 2 = 0$	Zero-Product Property
$x = 7$ or $x = -2$	Solve for x.

Check

$7 + 1 \stackrel{?}{=} \sqrt{7(7) + 15}$

$8 \stackrel{?}{=} \sqrt{64}$

$8 = 8$ ✓

$-2 + 1 \stackrel{?}{=} \sqrt{7(-2) + 15}$

$-1 \stackrel{?}{=} \sqrt{1}$

$-1 \neq 1$ ✗

▶ The apparent solution $x = -2$ is extraneous. So, the only solution is $x = 7$.

Homework

Pg 266, #3-20, 55